SOLUTIONS OF EXERCISES.

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A HOLLOW sphere, external and internal radii R and r, and specific gravity s, is partly filled with water, and floats in a pond, the water in the sphere being on a level with the surface of the pond. Find the quantity of water in the hollow sphere. [*Artemas Martin.*]

SOLUTION.

Let h be depth of the water in the sphere; then $h + R - r$ is the depth of sinking in the pond. If P, w, W be the weights of the water displaced, of the water in the sphere, and of the sphere respectively, then

$$P = w + W, \tag{1}$$

or

$$\pi(h + R - r)^2 \left[R - \tfrac{1}{3}(h + R - r)\right] = \pi h^2(r - \tfrac{1}{3}h) + \tfrac{4}{3}s\pi(R^3 - r^3).$$

This gives

$$h = \frac{4s(R^3 - r^3) + r(3R^2 - r^2) - 2R^3}{3(R^2 - r^2)} = \frac{4s(R^2 + Rr + r^2) - (2R + r)(R - r)}{3(R + r)};$$

whence the quantity of water in the sphere may be obtained from the expression $\pi h^2(r - \frac{1}{3}h)$. [*W. O. Whitescarver.*]

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A SUCTION pump is connected with a reservoir by a pipe of the same diameter as the piston. How many strokes are needed to bring water?

[*W. H. Echols.*]

SOLUTION.

Let the pipe be vertical, the lower side of the piston when at the upper end of the stroke be a feet above the surface of the reservoir, stroke of the piston b feet, the pressure of the atmosphere $p =$ a column of water 34 feet high, φ_1 the number of feet of water in the pipe at the end of the first stroke, φ_2 at end of second stroke, φ_n at end of nth stroke.

Then will the $a - b$ feet below the piston at the beginning of the first stroke be expanded to $a - \varphi_1$ feet at the end of the stroke, resulting in a pressure of air

$$p_1 = \frac{a - b}{a - \varphi_1} p.$$

This pressure p_1, added to the weight of the water in the pipe, will equal an atmosphere ; hence

$$\frac{a-b}{a-\varphi_1}p+\frac{\varphi_1}{p}=p\,.$$

Similarly, at the end of the second stroke,

$$\frac{(a-b)\,(a-b-\varphi_1)}{(a-\varphi_1)\,(a-\varphi_2)}\,p+\frac{\varphi_2}{p}=p,$$

from which φ_2 may be found. Finally, at the end of the nth stroke, we will have

$$\frac{(a-b)\,(a-b-\varphi_1)\ldots(a-b-\varphi_{n-1})}{(a-\varphi_1)\,(a-\varphi_2)\ldots(a-\varphi_n)}\,p+\frac{\varphi_n}{p}=p,$$

which will give φ_n. The number of strokes will be n when

$$\varphi_n > a-b,$$
$$\varphi_{n-1} < a-b.$$

[*De Volson Wood.*]

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FIND an approximate value for the perturbation of a comet by the sun, when the comet is very near a planet. (*See Watson's Astronomy, p.* 550).

SOLUTION.

Let M be the mass of the sun, r the distance of the sun from the planet, $\varDelta$ the distance of the comet from the planet, and θ the angle between r and $\varDelta$. Also let P be the disturbing force along $\varDelta$, and T the disturbing force perpendicular to $\varDelta$ in the plane of the comet's orbit. Denoting by R the perturbative, or potential function, taking only the first term of this function, and neglecting the perturbation in latitude which is of the second order, we have

$$R=\frac{M\varDelta^2}{2r^3}\,(1-3\cos^2\theta).$$

$$P=\frac{\partial R}{\partial\varDelta}=\frac{M\varDelta}{r^3}\,(1-3\cos^2\theta)\,,$$

$$T=\frac{\partial R}{\varDelta\partial\theta}=\frac{3\,M\varDelta}{r^3}\cos\theta\sin\theta.$$

$$\therefore\ \text{Resultant}=\sqrt{P^2+T^2}=\frac{M\varDelta}{r^3}\sqrt{1+3\cos^2\theta}.$$

The expression in Watson's Astronomy is erroneous, as was pointed out by Professor Lehmann-Filhes. [*A. Hall.*]